Enoch Yu

2014 KMO 2.

Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that the following equation holds for all $x, y \in \mathbb{R}$.

$$f(xf(x) + f(x)f(y) + y - 1) = f(xf(x) + xy) + y - 1$$

$f(x)=x$

1. $x=0$

$f(f(0)f(y)+y-1) = f(0)+y-1 \in \mathbb{R}$ → determed!

↳ surjective!

2. $\exists\ a \in \mathbb{R}$ s.t. $f(a)=0$ ↑

$f(y-1) = f(ay)+y-1$

$y-1=ay \Rightarrow y=\frac{1}{1-a} \Rightarrow a=1$?

3. $a=1$ cannot be the unique a?

FTSOC, $x=y=1 \Rightarrow f(0)=f(1)=0$

□

→ $y-1=0=\frac{a}{1-a}$

∵ If $a \neq 1$, then $a=0$.

$f(0)=0$

$(x=0)\quad f(y-1)=y-1$

∴ $\boxed{f(x)=x}$

□